Written Exam for the B.Sc. or M.Sc. in Economics autumn 2012-2013

Corporate Finance and Incentives

Correction Guide

Final Exam/ Elective Course/ Master's Course

20th December 2012

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title, which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The exam consists of 4 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different sub questions that do not necessarily have equal weight. Please provide intermediate calculations. Good luck ☺

Problem 1 (Various themes, 20%)

1. **What are the Macaulay Duration and the Modified Duration of a 5% annuity loan over 10 years and what do these numbers tell us?**

t	CF	PV	t * PV / V
1	12.95	12.33	0.12
2	12.95	11.75	0.23
3	12.95	11.19	0.34
4	12.95	10.65	0.43
5	12.95	10.15	0.51
6	12.95	9.66	0.58
7	12.95	9.20	0.64
8	12.95	8.77	0.70
9	12.95	8.35	0.75
10	12.95	7.95	0.80
SUM =		100.00	**5.10**

$$D_{Mac} = \frac{1}{PV(C,y)} \sum_{t=1}^{T} t \frac{c_t}{(1+y)^t}$$

The Macaulay Duration is 5.1 years, which tells us that there are 5.1 years on average to receiving the cash flows. It is also the price sensitivity relative to changes in (1+y).

The Modified Duration is: $D_{Mod} = D_{Mac} / (1+y)$ where y is the yield to maturity. This is the first derivative of the price with respect to the yield (y). This can be interpreted as an elasticity of how much the price of a bond changes in per cent when the yield changes one percentage point. D_{Mod} = 5.1 / 1.05 = 4.86, which means that if the yield increases by one percentage point the price drops by 4.86%.

2. **Explain what Convexity is, how we calculate it and what we use it for.**

Convexity measures how interest rate sensitivity (Duration) changes with the interest rate.

$$k = \frac{1}{PV(C,y)} \sum_{t=1}^{T} t^2 \frac{c_t}{(1+y)^t}$$

It is the curvature of the price-yield relation – the second derivative of the price function.

3. **Explain what a real option is, give a numerical example of a real option and calculate the value of the real option in your example.**

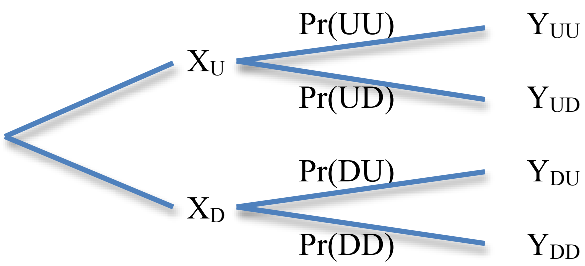
(Short explanation expected) An alternative or choice that becomes available with a business investment opportunity. Real options can include opportunities to expand and cease projects if certain conditions arise, amongst other options. They are referred to as "real" because they usually pertain to tangible assets such as capital equipment, rather than financial instruments. Taking into account real options can greatly affect the valuation of potential investments. Oftentimes, however, valuation methods, such as NPV, do not include the benefits that real options provide (Investopedia)

Students are expected to come up with a simple example and calculate the value of the real option. E.g.: A project has two phases. The first phase has a NPV=X and the second phase has a NPV=Y. The total value of the project is thus X+Y. The value NPV=Y is the expected value of phase two contingent of phase 1:

$NPV_{Phase2} = Y = Y_U + Y_D = [\ Pr(UU)\ Y_{UU} + Pr(UD)\ Y_{UD}\ ] + [\ Pr(DU)\ Y_{DU} + Pr(DD)\ Y_{DD}\ ]$
with:
$Y_U = [\ Pr(UU)\ Y_{UU} + Pr(UD)\ Y_{UD}\ ] > 0$, and $Y_D = [\ Pr(DU)\ Y_U + Pr(DD)\ Y_D\ ] < 0$

If phase 1 turns out well the expected value of phase 2 is positive but if phase 1 turns out bad the expected value of phase 2 is negative. The real option here can then be defined as: *The option to abandon the second phase of the project after observing the outcome of phase 1*. And the value of the real option is simply the expected value of phase 2 if phase 1 is not a success, $(-Y_D)$.

NPV of the full project is then: $NPV = X + Y + (-Y_D) = X + Y_U$



4. **Explain the put-call parity and explain what effect it has on a European put option on a non-dividend paying stock if the time to maturity of the option increases.**

Students are expected to briefly explain the intuition behind (stock + put = bond + call) and show the put-call parity in equation: $C = P + S - PV(K)$. The value of a put can be written as:

$$P = \underbrace{K - S}_{\text{Intrinsic value}} \underbrace{-dis(K) + C}_{\text{Time value}}$$

When time to maturity increases the value of the call option increases but at the same time the discounting effect on K – *dis(K)* – also increases leaving the final effect ambiguous.

Problem 2 (Factor Models and Arbitrage Pricing Theory, 20%)

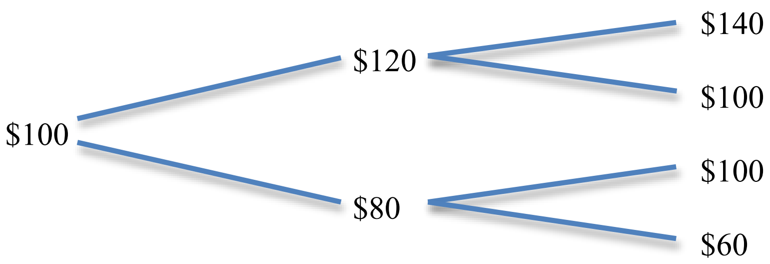
What are Factor Models and APT? In your explanation you could include (but should not feel limited to):

1. Assumptions
2. Describe what a factor model is
3. Types of independent variables in factor models
4. Comparison to CAPM
5. Give an example
6. Show what a pure factor model is
7. Show what an arbitrage is
8. Etc.

Problem 3 (Options, 20%)

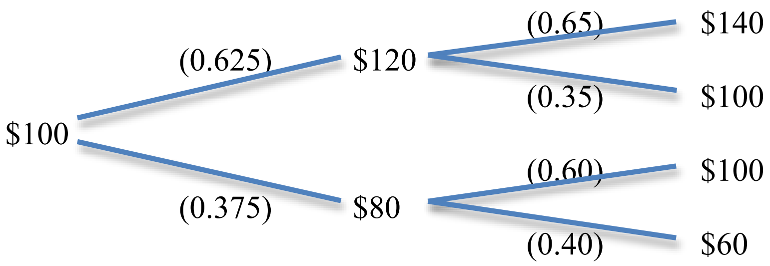
Consider a European call option on a non-dividend paying stock, which is currently trading at a price of \$100. Each period, the stock price can either go up by \$20 or down by \$20. Time to maturity is two periods and the risk free rate is 5% per period.

1. **Draw a binomial tree of above stock value distribution.**



2. **Find the risk-free probabilities for all periods and states.**

The risk free probabilities are found as: $\rho = \frac{(1 - r_f)S - S_d}{S_u - S_d}$, values inserted in the diagram below.



3. **What is the value of a call option with a strike price of \$120?**

C = 0.625 * 0.65 * \$(140 – 120) * $(1.05)^{-2}$ = \$7.37

4. **What is the value of the corresponding put option?**

P = C – S + PV(K) = \$7.37 - \$100 + 120 * $(1.05)^{-2}$ = \$16.21

Could alternatively be found by using the binary tree.

Problem 4 (Capital structure and the cost of capital, 40%)

Consider a firm with the expected future perpetual profit and loss statement (cash flows):

Turnover	**800**
Operating costs	**400**
EBITDA	**400**
Depreciation & Amortization	**100**
EBIT	**300**
Interest payments	**50**
Earnings before taxes (EBT)	**250**
Tax (20%)	**50**
Profit after tax	**200**

The firm pays 5% on debt, which can also be assumed to be the risk free rate. The market risk premium is 10% and you can assume that CAPM holds. The firm's equity beta is 1.5

1. **What is the expected return on equity and what is the market value of equity?**

According to CAPM the expected return on equity is: $E\left[r_E\right] = r_f + \beta_E \cdot MRP = 5\% + 1.5 \cdot 10\% = 20\%$.
The market value of equity is the sum of discounted expected future profits: E = 200 / 0.2 = 1000.

2. **What is the market value of debt?**

D = 50 / 0.05 = 1000

3. **What is the total value of the firm and what is the value of the un-levered firm?**

V^L: Value of levered firm
V^U: Value of un-levered firm
V^T: Value of tax shield = t * D (when debt is perpetual)

V^L = E + D = 2000
$V^U = V^L - V^T$ = 2000 – 20% * 1000 = 1800

4. **What is the value of the firm with maximum leverage?**

This is a trickier question than first seems. There are a few options of how to understand the question. Options are increasingly correct:

1) Equity of 1000 is switched to debt, adding another 200 worth of tax shield leaving the firm with a final value of **2200**.
2) Realizing that in scenario 1) equity would still be 200 (the added tax shield) after the switch and thus meaning an option to further adding leverage to the company. The value of the tax shield of those 200 is 40 leaving the firm with a value of **2240**.
3) This process is continued leaving the firm finally with (highly levered) value of:

$$V^L = 1800 + 1800 \cdot 20\% + 1800 \cdot 20\%^2 + \ldots = \frac{1800}{1-20\%} = \mathbf{2250}$$

5. **Explain the downside of leverage and why firms do not just increase debt close to 100%.**

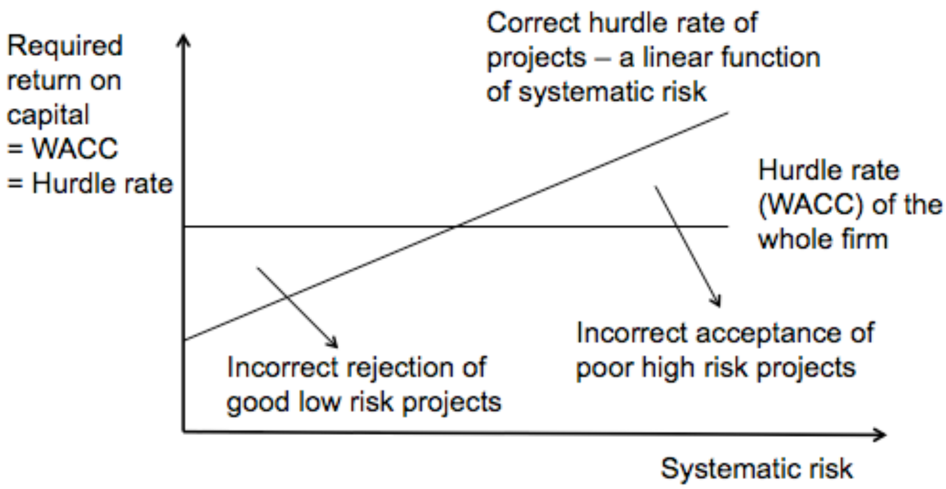
When the firm gets too highly levered debt becomes risky and debt holders will thus charge a risk premium. This is the cost of financial distress and occurs particularly when equity holders get strong incentives to shift risk to the debt holders. Keywords: *financial distress and risky debt*.

The firm is made up of two division serving two different segments. One division (A) is serving the government on long-term contracts and the other division (B) is serving other firms on a highly competitive market. Division A is considered safe but not risk free and division B is considered more risky. The two divisions approximately turnover the same amount per year.

6. **Comment on the above statement and in particular comment on what this means for the cost of capital in each of the two divisions.**

Cost of capital should reflect the systematic risk of projects. This means that since the two divisions are exposed to different systematic risks they should have different hurdle rates (WACCs). Division A serving the government should have a lower rate than division B being in a more risky industry.

7. **Draw a graph showing what would happen if the firm applies the same hurdle rate to all its projects disregarding division and explain the graph.**



There are not a lot of comparable firms in the industry for government services, but on the private side, there are a few other firms, which the firm considers close and similar competitors. The following information on the comparable firms is available:

	Expected return	Equity beta	D/(E+D)
Firm X	**25%**	**2**	**48%**
Firm Y	**18%**	**1.3**	**21%**
Firm Z	**22%**	**1.7**	**40%**

8. **What is the cost of capital for each of the two divisions?**

In order to derive the cost of capital for each department we must apply a few bits of information given and some general knowledge from finance theory:

1) We find the asset beta of the industry serving the private corporations by de-levering each of the comparable firms and taking the average. $\beta_A = \left(\frac{E}{E+D(1-t)}\right)\beta_E$ assuming risk free debt.
2) The total firm asset beta is the weighted average of the two divisional asset betas. Here weights are 50/50 since revenue is used as proxy for fixed assets. Asset beta for the whole firm is found just like under 1) by de-levering the equity beta.
3) Re-lever by applying the appropriate leverage: $\beta_E^L = \beta_A\left(\frac{E+D(1-t)}{E}\right)$

By 1) Calculations are done using Excel:

	Expected return	**Equity beta**	**D/(E+D)**	E/(E+D(1-t))	Asset beta
Firm X	**25%**	**2**	**48%**	0.58	1.15
Firm Y	**18%**	**1.3**	**21%**	0.82	1.07
Firm Z	**22%**	**1.7**	**40%**	0.65	1.11
Average					1.11

Average asset beta of the industry serving private corporations is thus 1.11

By 2) $\beta_A(\text{whole firm}) = \left(\frac{1000}{1000+1000(1-20\%)}\right)\cdot 1.5 = 0.83$

$$\beta_A(\text{whole firm}) = .5\cdot\beta_A(\text{Gov}) + .5\cdot\beta_A(\text{Priv})$$
$$\Updownarrow$$
$$\beta_A(\text{Gov}) = \frac{\beta_A(\text{whole firm}) - 5\cdot\beta_A(\text{Priv})}{.5} = \frac{.83-.5\cdot 1.11}{.5} = .56$$

By 3) Re-lever using a debt-to-equity rate of 50/50 and finally applying CAPM we get:

$$\beta_E^{Gov} = \beta_A^{Gov}\left(\frac{50+50(1-.2)}{50}\right) = .56\cdot\left(\frac{90}{50}\right) = 1.00 \Rightarrow E\left[r_E^{Gov}\right] = 5\% + 1\cdot 10\% = 15\%$$

and

$$\beta_E^{\text{Priv}} = \beta_A^{\text{Priv}}\left(\frac{50+50(1-.2)}{50}\right) = 1.11\cdot\left(\frac{90}{50}\right) = 2.00 \Rightarrow E\left[r_E^{Gov}\right] = 5\% + 2\cdot 10\% = 25\%$$